Enhancing the Natural Frequency Doublet Splitting in Almost Periodic Azimuthally Corrugated Cavities

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***Abstract*—A symmetry of a corrugated periodic structure can be broken due to loading or/and fabrication defects so that frequency doublets may appear, whose frequencies are just slightly splitted. In this letter, we study the potentials to enhance this splitting by enhancing the asymmetry of the cavity. This is provided by a single (loaded) resonator which differs in its geometry from other resonators. The initial causes of the asymmetry are not taken into account. The used characteristic matrix equation is obtained by a mode-matching technique. It is shown that the doublet splitting can reach several percent due to a convenient choice of the geometry of the loaded resonator.**

***Index Terms*—Azimuthally corrugated cavity, characteristic matrix equation, frequency doublet, frequency splitting.**

I. Introduction

THE MAIN motivation of the present study was the development of a new generation of millimeter-wave magnetrons operating at higher (for example, at the first backward) space harmonic, while the working mode is a non-π-mode [1]–[3]. In these devices, a waveguide-type output connected to one of the side resonators (loaded resonator) is used. In practice, the loaded Q-factor equals up to several hundred, so that one should expect that the doublet frequencies of each doublet differ by no more than 0.5%. Hence, the problem of a slight splitting of the frequency doublets, that occurs for those modes which are employed as working modes, appears due to the loading. Another possible cause of a slight splitting is slight defects in cavity fabrication.

The main goal of this letter is to demonstrate the potentials of an almost periodic cavity in order to avoid the slight splitting. We consider a cavity in which just the geometry of the loaded resonator is different from that of other resonators. So far as the expected frequency splitting should be about several percent, we consider neither the external loading effect nor any fabrication defects. The problem of enhancing the splitting, that results from fabrication defects, can occur for another application of azimuthally corrugated structures, for example, for quadruple-ridged circular waveguides [4]. The model proposed here can also be applied to study the potentials of doublet splitting for the above waveguide structures. In this paper, the emphasis in our numerical study is, however, on structures which show geometrical parameters which are typical for millimeter-wave magnetrons.

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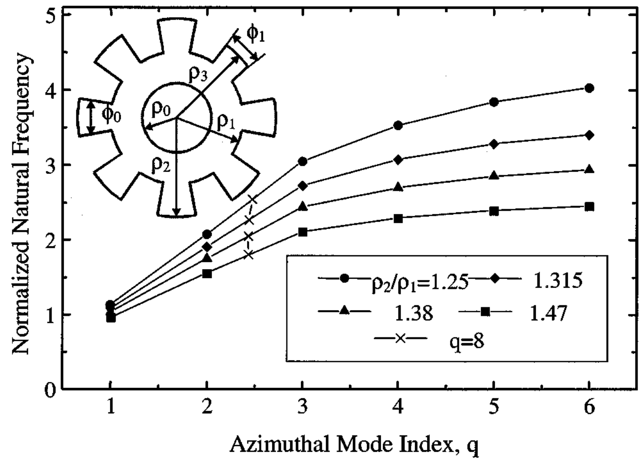

Fig. 1. Cross section of an almost periodic corrugated cavity and normalized natural frequencies of the cosine-polarized modes TE$_{q1}$ at $\rho_0/\rho_1 = 0.5$, $\vartheta = N\phi_0/2\pi = 0.55$, $N = 16$, $K_r = \rho_3/\rho_2 = 1.2$, $K_\phi = \phi_1/\phi_0 = 0.5$ and several values of ρ_2/ρ_1. The azimuthal mode index q is shown on the horizontal axis for $q = 1, 2, \ldots 6$. The frequencies at $q = 8$ are shown by a cross (x).

II. Theory

A cross section of the cavity is shown in the insert in Fig. 1. It has N side resonators and N ridges, where N is assumed to be an even number. We assume that the periodical location of the symmetry axes of the resonators remains unchanged so that the axes are located at $\phi_r = 2\pi r/N$, where $r = 0, 1, \ldots N-1$. The maximum radius and the aperture opening are given by ρ_2 and ϕ_0, respectively, for regular resonators, and by ρ_3 and ϕ_1, respectively, for the loaded one. ρ_0 is the radius of the smooth inner conductor. Ideal magnetic walls are assumed to be located at the end-faces of the cavity so that the natural frequencies corresponding to the axially homogeneous TE-modes of the cavity are equal to the cutoff wavenumbers of TE modes of a waveguide showing the same cross section.

A solution of the Helmholtz equation for each regular subregion of the cavity leads to the eigenmode field expansions, whose coefficients (amplitudes of the space harmonics) are not yet known. Using mode matching at $\rho = \rho_1$, we obtain the characteristic matrix equation

$$G_p - \theta \sum_{s=-\infty}^{\infty} G_s \beta_s^{-1}(k\rho_1, k\rho_0) \sum_{n=0}^{\infty} (2 - \delta_n^0) \cdot [\Phi^P(k) N \delta_{p+mN}^s + \Phi^{NP}(k)] = 0 \quad (1)$$

where G_p are the amplitudes of the space harmonics in exponential-function-based expansion of the eigenmode field in region

$\rho_0 \le \rho \le \rho_1$, δ_i^j is the Kronecker delta, $\theta = 1/2\pi\phi_0$, $k = \omega/c$, $p, m = 0, \pm 1, \pm 2, \ldots$

$$\Phi^P(k) = \tilde{a}_{np}\tilde{b}_{ns}\beta_{\nu_n}(k\rho_1, k\rho_2) \quad (2)$$

$$\Phi^{NP}(k) = \xi\tilde{a}_{np}^{(1)}\tilde{b}_{ns}^{(1)}\beta_{\gamma_n}(k\rho_1, k\rho_3) - \Phi^P(k) \quad (3)$$

$$\beta_\tau = \tilde{Z}_\tau(x_1, x_h)/Z_\tau(x_1, x_h). \quad (4)$$

In (2)–(4), $\nu_n = \pi n/\phi_0$, $\tilde{a}_{np} = -ip[(-1)^n \exp(-ip\phi_0) - 1](\nu_n^2 - p^2)^{-1}\exp(ip\phi_0/2)$, $\gamma_n = \pi n/\phi_1$, $\xi = \phi_0/\phi_1$, and the coefficients $\tilde{b}_{ns}$ are obtained from $\tilde{a}_{np}$ by replacing p by $-s$. The coefficients $\tilde{a}_{np}^{(1)}$ and $\tilde{b}_{ns}^{(1)}$ are then obtained from $\tilde{a}_{np}$ and $\tilde{b}_{ns}$ by replacing ϕ_0 by ϕ_1 and ν_n by γ_n, respectively. The functions Z_τ and $\tilde{Z}_\tau$ are given by

$$Z_\tau(x_1, x_h) = J_\tau(x_1) - Y_\tau(x_1)J'_\tau(x_h)/Y'_\tau(x_h)$$

and

$$\tilde{Z}_\tau(x_1, x_h) = \frac{d}{dz}Z_\tau(z, x_h)\Big|_{z=x_1}$$

where J_τ, Y_τ, J'_τ, and Y'_τ are the τ'th order Bessel and Neumann functions and their derivatives, respectively, $h = 0, 2, 3$.

In case of $\phi_1 = \phi_0$ and $\rho_3 = \rho_2$, $\Phi^{NP}(k) \equiv 0$, so that (1) is splitted into N independent matrix equations. Each of them coincides with the characteristic equation for one of the allowed values of the azimuthal index q of the azimuthally periodic cavity [5], $q + mN = s$, $q = -N/2 + 1, -N/2 + 2, \ldots 0, 1, \ldots N/2 - 1, N$. The mode with $q = N/2$ is known as the π-mode. If $\rho_2 = \rho_1$ and $\rho_0 = 0$, $\Phi^P(k) \equiv 0$. In this case, (1) represents the characteristic equation of the cutoff wavenumbers of the single-ridged waveguide [4]. No additional adaptation of (1) is needed for the double- and quadruple-ridged circular waveguides [4].

The structure shown in Fig. 1 has a single axis of symmetry, which coincides with the axis of symmetry of the loaded resonator. If N is rather large, one should expect that the symmetries of the periodic structure, which have been broken due to the loading, exert an effect on the natural frequency spectrum, too. Hence the spectrum in our case should show the features of the spectra of both single-ridged and periodic multi-ridged structures.

For computational convenience, we split (1) into two independent matrix equations, which correspond to the cases of cosine- and sine-polarizations. The splitting is possible due to the first-order symmetry of the considered structure. It has been done by using the unambiguous coincidence between exponential- and sine/cosine-function-based expansions of the eigenmode field in region $\rho_0 \le \rho \le \rho_1$, and the formula for the integer-order Bessel and Neumann functions that is $U_{-\tau}(x) = (-1)^\tau U_\tau(x)$ [6, p. 358], where τ means an integer and U_τ either J_τ or Y_τ. The normalized natural frequencies $k\rho_1$ are then calculated as zeroes of the truncated characteristic determinant, for each of the two polarizations.

III. Results

Figs. 1 and 2 show $k\rho_1$, corresponding to the cosine-polarized TE-modes, and a relative splitting factor η, respectively. η characterizes the extent to which the frequencies of each doublet are splitted. For convenience, we have used here that nomenclature of the modes, which is usually used for azimuthally corrugated periodic cavities [5]. The azimuthal index q for a mode of the almost periodic structure is assumed to be the same as that of that mode of the periodic structure which shows the same $k\rho_1$-value, if ρ_3 and ϕ_1 tend to ρ_2 and ϕ_0, respectively. However, one "jumps" over the ranges of a possible location of degeneracy and other critical points. Such a nomenclature is most appropriate to compare between the periodic and almost periodic structures. For both structures, there is a cosine-polarized mode, which should be related to the first passband, while a corresponding sine-polarized should be related to the second passband. In line with the used mode nomenclature, this mode is $TE_{N/2,1}$ in the periodic case. We will refer to the similar mode in the almost periodic case as to that with $q = N/2$, too.

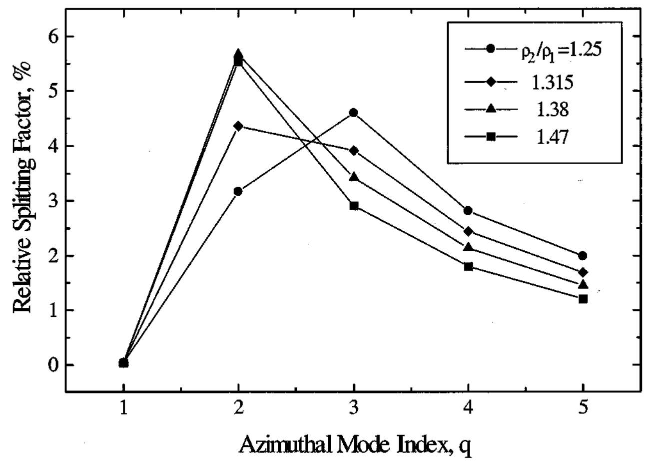

Fig. 2. Relative splitting factor η that is the relative difference between the natural frequencies of the cosine- and sine-polarized modes, for the same geometrical parameters as in Fig. 1.

At $q = 1, 2, \ldots N/2 - 1$, the only difference between the periodic and almost periodic cases is that for each pair of the cosine- and sine-polarized modes, $k\rho_1$-values are equal in the former case but differ in the latter. Both the cosine- and sine-polarized modes are related to the first passband for these q.

It can be seen in Fig. 2 that the strongest frequency splitting at $q = \text{const}$ occurs at $q = R, R+1$, which satisfy the inequality

$$k_R < k_{N/2} < k_{R+1}. \quad (5)$$

Fig. 3 shows $k\rho_1$ for $q = N/2$ for several cavities. Selecting a value of $K_r = \rho_3/\rho_2$, one can provide fulfillment of (5) for any R, $1 \le R \le N/2 - 2$. The $k\rho_1$-value at $q = N/2$ much stronger depends on K_r than on K_ϕ. Besides, it much stronger depends on K_r than that at $q \ne N/2$ so that the $k\rho_1$-value is mainly influenced by the only nonbroken symmetry. The efficiency of the proposed way to enhance the splitting can be lost if the density of the frequency spectrum for each of two polarizations is too high. Then one should use a cavity with smaller N and ρ_2/ρ_1 in order to decrease the density.

The values of $k\rho_1$ and η are given in Table I for several modes and cavities, thus demonstrating additional potentials to increase η. The cavity A in the Table I shows the following geometrical parameters: $\rho_0/\rho_1 = 0.5$, $\rho_2/\rho_1 = 1.25$, $\vartheta = 0.55$, and $N = 16$. For the cavity B, we have set $\rho_0/\rho_1 = 0.5$,

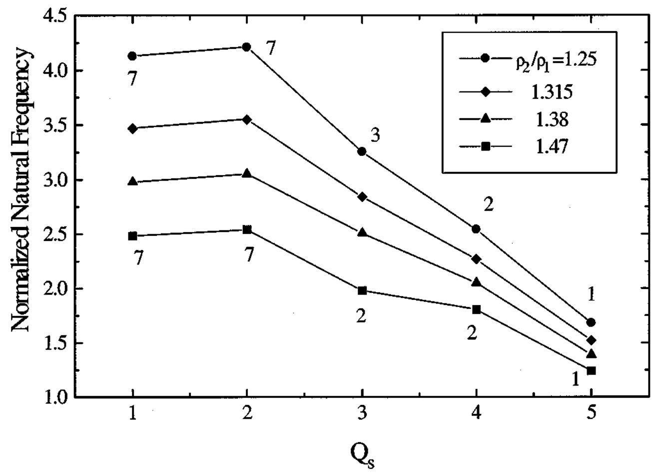

Fig. 3. Normalized natural frequencies for the cosine-polarized mode $TE_{N/2,1}$ for cavities with different geometrical parameters of the loaded resonator: $K_r = 1.0$ at $Q_s = 1, 2$, K_r is equal to 1.1, 1.2, and 1.4 at $Q_s = 3$, 4, and 5, respectively; $K_\phi = 1.0$ at $Q_s = 1$ and 0.5 otherwise; the values of ρ_0/ρ_1, N, and ϑ are the same as in Fig. 1. A number at the signs showing the frequencies, S, means the largest q-value, for which the $k\rho_1$-eigenvalue is smaller than that at $q = N/2$; in cases of $\rho_2/\rho_1 = 1.315$ and $\rho_2/\rho_1 = 1.38$, the S-value is equal to that in case of $\rho_2/\rho_1 = 1.25$ for each Q_s.

TABLE I
SEARCHING FOR POTENTIALS TO MAXIMIZE THE SPLITTING OF THE FREQUENCY DOUBLETS: THE η-VALUE FOR SEVERAL CAVITIES AND MODES, FOR WHICH THE SPLITTING IS STRONGLY PRONOUNCED

Cavity	K_r	K_ϕ	q/S	$k\rho_1$	η, %
A	1.1	1.6	1/3	1.102	3.56
A	1.1	1.6	3/3	2.724	7.22
B	1.0	0.5	1/3	0.706	4.51
B	1.2	0.5	2/1	1.096	6.15
B	1.4	0.5	1/1	0.594	13.6
B	1.4	0.5	2/1	1.084	5.45

$\rho_2/\rho_1 = 1.9$, $\vartheta = 0.71$, and $N = 8$. As can be seen, if N is small enough, η can reach several percent even if $K_r = 1$. The use of a cavity with $K_\phi > 1$ also results in enhancing the doublet splitting even if ρ_3/ρ_2 is rather small.

All results given in Figs. 1–3 and in Table I have been obtained, while 5 space harmonics in region $\rho_1 \le \rho \le \rho_2$ are taken into account. A number of the harmonics in region $\rho_0 \le \rho \le \rho_1$ has been chosen by taking into account the relative convergence phenomenon. Table II demonstrates the accuracy of simulations while a ratio of the numbers of the harmonics is close to that which is optimum from the point of view of the relative convergence. $\max|m|$and $\max n$ show the maximum values of $|m|$ and n occuring in the matrix, respectively. The use of a larger number of the harmonics as compared to that used to obtain the results in Table II does not lead to any substantial variation in

TABLE II
$k\rho_1$-EIGENVALUES OBTAINED BY USING SEVERAL TRUNCATED CHARACTERISTIC MATRICES FOR THE CAVITY WITH $\rho_2/\rho_1 = 1.25$ AND THE SAME OTHER PARAMETERS AS IN FIG. 1

$max\|m\|/$ $max\, n$	16/1	58/4	72/5
cosine-polarization			
q=2	2.0809	2.0759	2.0758
8	2.5710	2.5422	2.5401
3	3.0507	3.0486	3.0474
4	3.5186	3.5278	3.5253
sine-polarization			
2	2.1398	2.1413	2.1416
3	2.9029	2.9084	2.9077
4	3.4186	3.4288	3.4264

$k\rho_1$-eigenvalue in a wide range of the variation of the geometrical parameters.

IV. CONCLUSIONS

In this paper, the peculiar features of the natural frequency spectrum have been studied for the first passband of the almost periodic cavity, whose loaded resonator is distinguished from the other resonators in its depth and aperture opening. It has been shown that a relative splitting of the frequency doublet can be provided at a value of several percent. The proposed way to enhance the splitting is most promising for a cavity having rather small values of the number and depth of resonators.

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